Topic: First derivative test

Question: Use the First Derivative Test to find relative extrema.

$$f(x) = x^4 - 16x^2$$

Answer choices:

A Absolute minimum at $\left(\pm\sqrt{8}, -64\right)$; Relative maximum at $(0,0)$

B Relative minimum at $\left(-\sqrt{8}, -64\right)$; Relative maximum at $\left(\sqrt{8},64\right)$

C Relative minimum at $\left(\sqrt{8}, -64\right)$; Relative maximum at $\left(-\sqrt{8},64\right)$

D No absolute minimum; Relative maximum at $(0,0)$

© Krista King Math

Solution: A

To find the relative extrema of the function, we first need to find the critical numbers. This is done by setting the first derivative of the function equal to 0 and solving for the variable.

$$f'(x) = 4x^3 - 32x$$

$$4x^3 - 32x = 0$$

$$4x\left(x^2 - 8\right) = 0$$

$$4x = 0$$

$$x = 0$$

or

$$x^2 - 8 = 0$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

We now use the critical numbers to divide the domain into intervals and determine the intervals where the function is increasing and decreasing. We can then determine whether our critical numbers are maxima, minima, or neither.

Interval	$\left(-\infty, -2\sqrt{2}\right)$	$\left(-2\sqrt{2}, 0\right)$	$\left(0, 2\sqrt{2}\right)$	$\left(2\sqrt{2}, \infty\right)$
Test value	$x = -3$	$x = -1$	$x = 1$	$x = 3$

© Krista King Math

$f'(x)$	$4(-3)^3 - 32(-3)$	$4(-1)^3 - 32(-1)$	$4(1)^3 - 32(1)$	$4(3)^3 - 32(3)$
	-12, Negative	28, Positive	-28, Negative	12, Positive
$f(x)$	Decreasing	Increasing	Decreasing	Increasing

So our conclusion is

Absolute minimum at $\left(-\sqrt{8}, -64\right)$

Relative maximum at $(0,0)$

Absolute minimum at $\left(\sqrt{8}, -64\right)$

© Krista King Math

Topic: First derivative test

Question: What are the extrema of the function using the first derivative test?

$$f(x) = x^2 - 2x - 1$$

Answer choices:

A There is a global minimum at the point $(-2,1)$.

B There is a global maximum at the point $(-2,1)$.

C There is a global minimum at the point $(1,-2)$.

D There is a global maximum at the point $(1,-2)$.

© Krista King Math

Solution: C

To find the critical numbers, take the first derivative of the function

$$f(x) = x^2 - 2x - 1$$

$$f'(x) = 2x - 2$$

Now set the derivative equal to 0 and solve for x.

$$0 = 2x - 2$$

$$2x = 2$$

$$x = 1$$

There is one critical number at $x = 1$. Now take a point on either side of $x = 1$ (we will use $x = 0$ and $x = 2$) and substitute these into the first derivative.

$$f'(0) = 2(0) - 2$$

$$f'(0) = -2$$

and

$$f'(2) = 2(2) - 2$$

$$f'(2) = 4 - 2$$

$$f'(2) = 2$$

© Krista King Math

We can see that on the left-hand side of $x = 1$ the function is negative, so it's decreasing. On the right-hand side of $x = 1$ the function is positive, so it's increasing.

Since the function at the point $x = 1$ goes from decreasing to increasing, a minimum exists at this point. This point is a global minimum because there is no lower point on the function.

Finally, we'll plug $x = 1$ into the original function to get the corresponding y -value.

$$f(x) = x^2 - 2x - 1$$

$$f(1) = (1)^2 - 2(1) - 1$$

$$f(1) = 1 - 2 - 1$$

$$f(1) = -2$$

This means that the point is $(1, -2)$. So the function $f(x) = x^2 - 2x - 1$ has a global minimum at the point $(1, -2)$.

© Krista King Math

Topic: First derivative test

Question: What are the extrema of the function using the first derivative test?

$$f(x) = \frac{1}{2}x^4 + \frac{2}{3}x^3 - 2x^2 + 7$$

Answer choices:

A There is a local minimum at the point $\left(-2, \frac{5}{3}\right)$, a local maximum at the point $(0,7)$ and a global minimum at the point $\left(1, \frac{37}{6}\right)$.

B There is a global minimum at the point $\left(-2, \frac{5}{3}\right)$, a local maximum at the point $(0,7)$ and a local minimum at the point $\left(1, \frac{37}{6}\right)$.

C There is a local maximum at the point $\left(-2, \frac{5}{3}\right)$, a local minimum at the point $(0,7)$ and a global maximum at the point $\left(1, \frac{37}{6}\right)$.

D There is a global maximum at the point $\left(-2, \frac{5}{3}\right)$, a local minimum at the point $(0,7)$ and a local maximum at the point $\left(1, \frac{37}{6}\right)$.

© Krista King Math

Solution: B

To find the critical numbers, take the first derivative of the function

$$f(x) = \frac{1}{2}x^4 + \frac{2}{3}x^3 - 2x^2 + 7$$

$$f'(x) = 2x^3 + 2x^2 - 4x$$

$$f'(x) = 2x\left(x^2 + x - 2\right)$$

$$f'(x) = 2x(x + 2)(x - 1)$$

Now set the derivative equal to 0 and solve for x.

$$0 = 2x(x + 2)(x - 1)$$

$$x = -2,\ 0,\ 1$$

There are three critical numbers at $x = -2,\ 0,\ 1$. Now take a point in each interval (we will use $x = -3$, $x = -1$, $x = 1/2$, and $x = 2$) and substitute these into the first derivative.

$$f'(-3) = 2(-3)(-3 + 2)(-3 - 1)$$

$$f'(-3) = -24$$

and

$$f'(-1) = 2(-1)(-1 + 2)(-1 - 1)$$

$$f'(-1) = 4$$

and

© Krista King Math

$$f'\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)\left(\frac{1}{2}+2\right)\left(\frac{1}{2}-1\right)$$

$$f'\left(\frac{1}{2}\right) = -\frac{5}{4}$$

and

$$f'(2) = 2(2)(2+2)(2-1)$$

$$f'(2) = 16$$

We can see that on the left-hand side of $x = -2$ the function is negative, so it's decreasing. Between $x = -2$ and $x = 0$, the function is positive, so it's increasing. Between $x = 0$ and $x = 1$, the function is negative, so it's decreasing. On the right-hand side of $x = 1$, the function is positive, so it's increasing.

Since the function at the point $x = -2$ goes from decreasing to increasing, a minimum exists at this point. Since the function at the point $x = 0$ goes from increasing to decreasing, a maximum exists at this point. Since the function at the point $x = 1$ goes from decreasing to increasing, a minimum exists at this point.

Finally, we'll plug $x = -2, 0, 1$ into the original function to get the corresponding y-values.

$$f(x) = \frac{1}{2}x^4 + \frac{2}{3}x^3 - 2x^2 + 7$$

$$f(-2) = \frac{1}{2}(-2)^4 + \frac{2}{3}(-2)^3 - 2(-2)^2 + 7 = \frac{5}{3}$$

© Krista King Math

$$f(0) = \frac{1}{2}(0)^4 + \frac{2}{3}(0)^3 - 2(0)^2 + 7 = 7$$

$$f(1) = \frac{1}{2}(1)^4 + \frac{2}{3}(1)^3 - 2(1)^2 + 7 = \frac{37}{6}$$

This means that the point $(1,37/6)$ is the local minimum, that $(-2,5/3)$ is a global minimum, and that $(0,7)$ is the local maximum.

© Krista King Math